The Parabolic Probability Distribution
Modeling Random Share Price

Gary Schurman, MBE, CFA

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In Part I we created our own probability distribution via a simple polynomial equation. In this white paper we will use the mathematics from that distribution to model the random share price at some future time T given the known share price at time zero. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

Our client owns shares in a startup where the current share price is \$25.00. We are tasked with calculating our current share price multiple at some future time T given the following go-forward assumptions...

Table 1: Modeling Assumptions

Assumption	Value	Notes
Probability that multiple $= 0.00$	0.30	If multiple is zero then startup fails at time T.
Probability that $0.00 <$ multiple ≤ 2.00	0.20	Arbitrary data point in $[0, 6]$ range.
Probability that $2.00 <$ multiple ≤ 6.00	0.50	Maximum price at time T is six times current price.

Question 1: What is the probability that the share price multiple will be in the range $[3.00, 4.50]$ at time T?
Question 2: Assume that the random number pulled from a uniform distribution is 0.75, what is the value of the corresponding random share price multiple at time T?

Probabiliy Distribution

We defined $f(x)$ to be our cumulative probability distribution for the random share price multiple (multiple of known share price at time zero) at time T. The equation for our parabola from Part I is... [1]

$$f(x) = a\,x^3 + b\,x^2 + c\,x + d \text{ ...where... } a = -0.0055, b = 0.0481, c = 0.0258, d = 0.3000 \text{ ...and... } 0 \leq x \leq 6 \tag{1}$$

Per Equation (1) above, the minimum multiple is 0.00 and the maximum multiple is 6.00. If the multiple is one then share price at future time T will be the same as share price at time zero (i.e. today). If the multiple is zero then the startup fails and the shareholder is wiped out sometime over the time interval $[0, T]$.

Using Equation (1) above, the first and second derivatives of that parabola with respect to the independent variable x are...

$$f'(x) = 3\,a\,x^2 + 2\,b\,x + c \text{ ...and... } f''(x) = 6\,a\,x + 2\,b \tag{2}$$

The equations for the first and second moments of our distribution are... [1]

$$FM = \frac{3}{4}\,a\,w^4 + \frac{2}{3}\,b\,w^3 + \frac{1}{2}\,c\,w^2 = 2.05 \text{ ...and... } SM = \frac{3}{5}\,a\,w^5 + \frac{2}{4}\,b\,w^4 + \frac{1}{3}\,c\,w^3 = 7.40 \tag{3}$$

Using Equation (3) above, the equations for the mean, variance, and standard deviation of our probability distribution are... [1]

$$\text{Mean} = FM = 2.05 \text{ ...and... } \text{Variance} = SM - FM^2 = 3.19 \text{ ...and... } \text{Std Deviation} = \sqrt{\text{Variance}} = 1.79 \tag{4}$$

Modeling Future Share Price

We will define the variable S_T to be random share price at time T and the variable θ to be the share price multiple (a random variable) at time T. The equation for random share price at time T as a function of known share price at time zero is...

$$S_T = \theta\, S_0 \;\;\text{...where...}\;\; S_0 \text{ is known at time zero and } \theta = 1 \text{ at time zero} \tag{5}$$

Using Equation (1) above, the equations for the probability that the random share price multiple will be zero at time T and less than or equal to w at time T are...

$$\text{Prob}\Bigg[\theta = 0\Bigg] = f(0) = d \;\;\text{...and...}\;\; \text{Prob}\Bigg[\theta \le w\Bigg] = f(w) = 1.00 \tag{6}$$

Using Equation (1) above, the equation for the probability that the random share price multiple will be in the range $[m < \theta < n]$ at time T is...

$$\text{Prob}\Bigg[m < \theta < n\Bigg] = f(n) - f(m) = a\,(n^3 - m^3) + b\,(n^2 - m^2) + c\,(n - m) \;\;\text{...where...}\;\; 0 \le m < n \le w \tag{7}$$

To simulate random share price at time T we want to pull random share price multiples from our probability distribution. Our first step is to pull a random number from a uniform probability distribution...

$$z = \text{Random number pulled from a uniform distribution with range } [0,1] \tag{8}$$

Note that the random number z in Equation (8) above can be viewed as a cumulative probability. To get our random share price multiple we take that random cumulative probabiliy and solve for the independent variable x. This statement in equation form is...

$$\text{If random number } z = a\,x^3 + b\,x^2 + c\,x + d \text{ then we want to solve that equation for share price multiple } x. \tag{9}$$

To solve for the share price multiple x in Equation (9) above we iterate the following Newton-Raphson equation until x and $\hat{x}$ are approximately equal...

$$\text{new } \hat{x} = \hat{x} + \frac{f(x) - f(\hat{x})}{f'(\hat{x})} \;\;\text{...where...}\;\; \hat{x} = \text{estimate of actual } x \;\;\text{...and...}\;\; f(\text{actual } x) = z \tag{10}$$

The Answers To Our Hypothetical Problem

Question 1: What is the probability that the share price multiple will be in the range $[3.00, 4.50]$ at time T?

Using Equations (1) and (7) above, the answer to the question is...

$$\text{Prob}\Bigg[3.00 < \theta < 4.50\Bigg] = -0.0055 \times (4.50^3 - 3.00^3) + 0.0481 \times (4.50^2 - 3.00^2) + 0.0258 \times (4.50 - 3.00) = 0.1592 \tag{11}$$

Question 2: Assume that the random number pulled from a uniform distribution is 0.75, what is the value of the corresponding random share price multiple at time T?

xhat	f(x)	f(xhat)	f'(xhat)
2.0525	0.7500	0.5080	0.1538
3.6261	0.7500	0.7640	0.1579
3.5376	0.7500	0.7499	0.1599
3.5382	0.7500	0.7500	0.1599
3.5382	0.7500	0.7500	0.1599

Note that our guess value (row one) is the mean of our distribution (Equation (4) above).

The answer to the question is 3.54.

References

[1] Gary Schurman, *The Parabolic Probability Distribution - Building Our Distribution,* September, 2023.